ONE MECHANISM OF INTERACTION OF AN ELASTIC WALL WITH A FLOW

V. I. Korobov and V. V. Babenko

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Results are presented from measurement of the drag coefficient in flow about elastic plates. Results of measurement of the dynamic characteristics of the plates are also presented.

It has been established that the application of a layer of an elastic material to the surface of a body may improve conditions for the flow of a liquid about the body (an extensive bibliography is presented in [1]). Studies [1] have been conducted under different conditions and on different types of coatings (mainly of the membrane type), and, in certain cases, the reverse effect has been observed. The character of the interaction of the elastic boundary with the flow remains unresolved. Theoretical studies have focused mainly on problems of laminarization of the boundary layer. In [2, 3, etc.], investigators also solved the problem of turbulent flow over an elastic surface.

Presented below are results from an experimental study of the integral characteristics of the boundary layer on monolithic elastic surfaces made in the form of a composition consisting of layers of polyurethane foams (PTF). These results, together with measured dynamic characteristics of the elastomers used, make it possible to more purposefully approach the given problem.

The tests were conducted within a broad range of Reynolds numbers, so the elastic surfaces were studied in a water tunnel (experiment I, Fig. 1, curves 4-6; the dashed line shows part of the experimental curves with a correction for the systematic error at $0.9 \cdot 10^6 < \mathrm{Re} < 1.4 \cdot 10^6$) and in a water channel [4] (experiment II, curves 7-12). Elastic inserts of thickness h in the form of rectangular panels ($a:l_e = 1:2.12$) were fastened on a tensometric suspension along the sides of a plate flush with the surface over which flow was taking place. The plate had fairings on the front and rear and end rings and was towed in the vertical position. Here, we measured the total fluid friction C_F acting on the outer surface of both panels. A swirl vane was installed ahead of the plate. The experimental set-up was shown in [4] ($L = l_0 + l_e$, where l_0 is the distance from the front edge of the plate to the elastic insert).

The drag relation of a standard rigid plate (curves 4 and 7) made of polished organic glass agrees well with the familiar Prandtl—Schlichting law (curve 3) for turbulent flow. Curves 1 and 2 correspond to the drag for laminar flow and for the transition from laminar to turbulent flow.

The relations obtained (curves 5, 6, 8-12) indicate the selective nature of the effect of elastic surfaces with uncontrolled mechanical parameters on the boundary layer, i.e., there is a certain Reynolds number range in which the hydrodynamic effect ξ is maximal, with the ranges of Re and ξ here depending on the viscoelastic properties of the surface. This situation is quite different from the familiar (M. O. Kramer and R. D. Galway) empirical curves of $C_F(\mathrm{Re})$, which have an optimum of opposite sign.

The results can be explained by the fact that the viscoelastic boundary, interacting with the incoming flow, absorbs some of its pulsative energy and thereby prevents the generation of secondary turbulence. The amount of energy absorbed by the layer is inversely proportional to its stiffness; in turn, only some of the absorbed energy, proportional to the loss factor, is dissipated in the layer.

It is known that a number of the mechanical properties of elastomers depend on the frequency of the acting load. In this connection, we measured the dynamic characteristics of the elastomers we used under cyclic loading by the method of forced nonresonant vibrations

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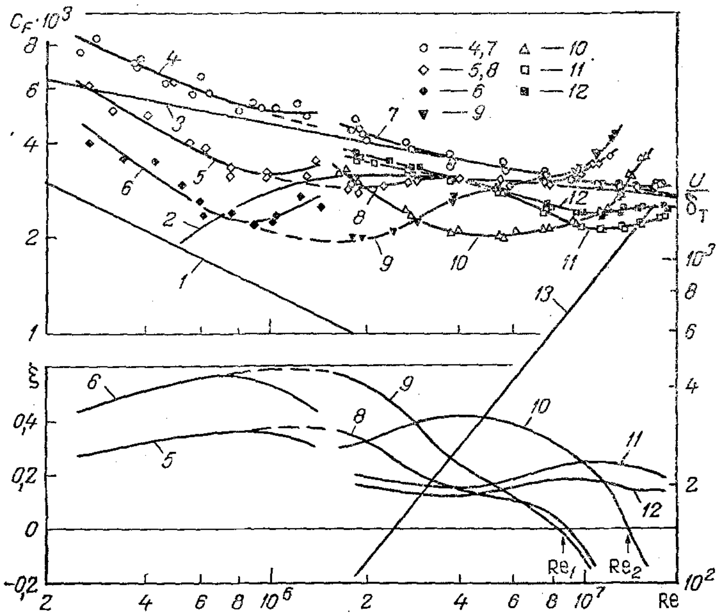

Fig. 1. Drag coefficient $C_F(Re)$ of stiff (1-4, 7) and elastic plates and their drag reduction factor $\xi(Re)$ (5, 6, 8-12) at different Reynolds numbers: 4-6) tests on hydraulic stand; 7-12) towing of plates in water channel [4]; 5, 8) PPU-1, h/L = 0.005; 6, 9) PPU-1, saturated with water, h/L = 0.005; 10) PPU-2, h/L = 0.004; 11) PPU-3, h/L = 0.006; 12) PPU-3, h/L = 0.003; 13) range of energy-carrying frequencies U/δ_t of the pulsative load in the boundary layer of the plate in relation to Re_L at x = L. U/δ_t, sec^{-1}.

[5]. We developed a special unit for this purpose. In the experiment, we determined the complex modulus of elasticity $\vec{E} = \vec{E}' + \vec{E}''$ and the angle of phase shift φ between the components E' and E'' as a function of the frequency ω; $\tan\varphi = E''/E'$ is the loss factor for the energy dissipated in the material on elastic flaws. Values of $E'(\omega)$ (solid curves) and $\tan\varphi(\omega)$ (dashed curves) are shown in Fig. 2.

Figure 1 (curve 13) shows the relation $U/\delta_t(Re)$, which indicates the upper boundary of the range of energy-carrying frequencies ($0 < \omega/2\pi < U/\delta_t$) of the spectrum of turbulent pulsations for each value of Re. It was assumed that the spectral density changes little within this interval and can be considered constant. Here, U is the velocity corresponding to the chosen Re number; δ_t is the thickness of the turbulent boundary layer, calculated from the formula $\delta_t(x) = 0.37x\, Re_x^{-0.2}$ at x = L.

It can be seen from Figs. 1 and 2 that there is a correlation between $tg\varphi(\omega)$ and the dynamic stiffness $C(\omega)$ at the corresponding frequencies and the magnitude of the effect for different plates. (The thickness h of the elastic plates, designated as 5-11 in Fig. 1, was roughly the same.) Moreover, for all of the elastic plates, there is a correspondence between the frequency of the maximum values of $\tan\varphi$ and the upper value of the frequency range of the energy-carrying part of the pulsation spectrum when the hydrodynamic effect is maximal (as a first approximation, we may assume that $\omega|_{\tan\varphi=\max} \approx U/\delta_T|_{\xi=\max}$). In this case, we see the greatest spectral density of the frequencies, with a maximum of the energy loss factor in the wall material within the limits of the energy-carrying part of the pulsative load spectrum, i.e., there is a clear correlation between the spectrum of boundary layer pulsations and the spectrum of the relaxation times of the elastic wall. This serves as proof of the existence of the mechanism of energetic interaction of the pliable surface with the boundary layer. As the Re number increases ($Re > Re_{\xi=\max}$), such as for PPU-2, (Fig. 1, curve 10), $R > 4 \cdot 10^6$, the range of energy-carrying frequencies broadens and the proportion of frequencies in the spectrum at which maximum dissipation of vibration energy occurs in the elastomer decreases, so that there is a decrease in the hydrodynamic effect.

The value of ξ should monotonically decrease with an increase in Re, as was seen, for example, for plates 8 and 9 at $3.5 \cdot 10^6 < Re < 7 \cdot 10^6$. However, ξ then decreases sharply and at $Re_1 = 8.55 \cdot 10^6$ $\xi = 0$. Similarly, for 10, $\xi = 0$ at $Re_2 = 1.39 \cdot 10^7$.

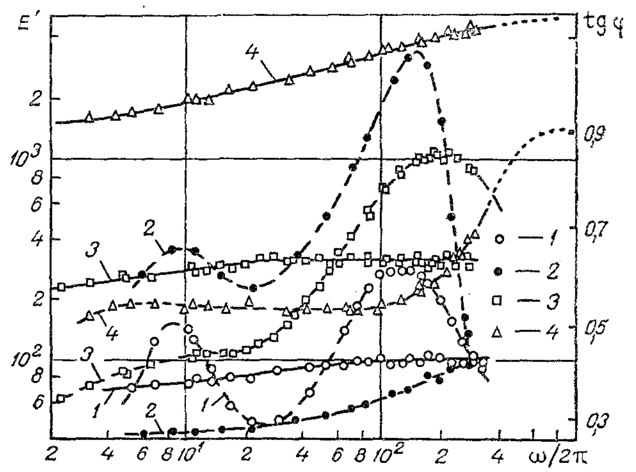

Fig. 2. Dependence of the viscoelastic properties of the elastomers on frequency: 1) PPU-1, ρ = 29 kg/m^3; 2) PPU-1, saturated with water; 3) PPU-2, ρ = 33 kg/m^3; 4) PPU-3, ρ = 1250 kg/m^3. E', kPa; $\omega/2\pi$, sec^{-1}.

Proceeding on the basis of the condition of ensuring hydraulic (dynamic) smoothness with allowance for the real thickness h_i of the elastomeric plates, we calculated the minimum permissible elastic modulus E_{min} of the wall material with the indicated numbers Re_1 and Re_2 using the formula:

$$E_{min}/h = C_{min} = 0.005\rho U^3 \nu^{-1} \mathrm{Re}^{-0.3}.$$

In the calculation, we used the criterion of the permissible height of the grainy surface roughness $Re_{k_{per}}$ = 100. The mean value of the pressure pulsations throughout the band of energy-carrying frequencies was determined from the formula $(\bar{p}^2)^{1/2} = 0.5\rho U^2 \times \mathrm{Re}^{-0.3}$. In both cases, the calculated value of E_{min} turned out to be 1.5 times greater than the measured value $E_i'(\omega)$ (Fig. 2) at the frequencies corresponding to Re_1 and Re_2 for PPU-1 and PPU-2 (i.e., $E_{min}/E'_{i_{measr}}$ = 1.5, with a difference of 2%). The latter fact may serve as quantitative confirmation of the energy character of the interaction of the elastic boundary with the turbulent boundary layer. The fact that the limiting permissible roughness on the elastic boundary is 1.5 times greater than that for the rigid boundary does not contradict the physical picture of flow over the surface, since it is quite natural to assume that a wavy roughness — not a granular roughness — will be formed on the elastic surface as the level of the pulsative load increases.

NOTATION

Re = UL/ν, Reynolds number; C_F, drag coefficient; $\xi = (C_{F_{measur}} - C_{F_{measur}}) C_{F_{measur}}^{-1}$, drag reduction factor; h, a, l_e, thickness, width, and length of the elastic plates; $\vec{E}$, complex modulus of elasticity, $\vec{E} = \vec{E}' + \vec{E}''$; E', E'', elastic and viscous components of elastic modulus; tan φ = E''/E', energy loss factor; C = E'/h, stiffness of the elastic wall in compression; ω, angular frequency of the pulsative load; δ_t, thickness of the turbulent boundary layer; U, velocity of the incoming flow; ρ, ν, density and kinematic viscosity; k_{per}, permissible height of granular roughness; PPU (PTF), polyurethane foam.

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CALCULATION OF THE COOLING OF A LIQUID MOVING IN AN UNDERGROUND CHANNEL

V. A. Makagonov, V. M. Sal'nikov, and V. M. Lukin UDC 536.242

An approximate solution is obtained for a problem of the cooling of a liquid moving a channel. The problem is solved on the basis of the use of the Laplace transform and the variational method.

The first solution to the problem of the cooling of a liquid in an underground passage was first given by Van-Heerden [1]. The classical method of the Laplace transform was used to obtain the solution. The final expression for the change in the temperature of the liquid in the channel was a complex relation which included Bessel functions of the first and second kind. Practical realization of this expression presented certain difficulties, even with the aid of a computer.

It should be noted that, in all cases, use of integral transforms over time leads to solutions in the form of infinite functional series or improper integrals. Here, only the main part of these expressions is used for practical calculations. Thus, if a simple method is found for directly determining a function equivalent to the main part of the exact solution, then it may be justifiably considered an approximate method suitable for practical application. Such a method, based on the joint use of the Laplace transform and the variational method, was proposed by Tsoi [2].

Let us examine the solution of the Van-Heerden problem by this approximate method and show how much simpler the final expression for the temperature of the liquid moving in the channel appears and how much more convenient it is for practical purposes.

The mathematical formulation of the problem in generalized variables has the form

$$\frac{\partial\eta}{\partial\tau'} = a\left[\frac{\partial^2\eta}{\partial r^2} + \frac{1}{r}\frac{\partial\eta}{\partial r}\right], \tag{1}$$

$$Q\frac{\partial\Theta}{\partial x} = \lambda_{\mathrm{gr}}\frac{\partial\eta}{\partial r}\bigg|_{r=R}. \tag{2}$$

The boundary conditions

$$\Theta = \eta \quad \text{at} \quad r = R, \tag{3}$$

$$\eta = 0 \quad \text{at} \quad r = \infty, \tag{4}$$

$$\Theta = 1 \quad \text{at} \quad r = 0. \tag{5}$$

The initial conditions

$$\left.\begin{aligned}\Theta = 0\\ \eta = 0\end{aligned}\right\} \quad \text{at} \quad \tau' = 0, \tag{6}$$

where

$$\Theta = \frac{t_{\mathrm{a}} - t_{\mathrm{gr}}}{t_{0\mathrm{v}} - t_{0\mathrm{gr}}}; \quad \eta = \frac{t_{\mathrm{gr}} - t_{\mathrm{gr}}}{t_{0\mathrm{v}} - t_{0\mathrm{gr}}}; \quad \tau' = \tau - \frac{x}{V}; \quad Q = 0.5c_{\mathrm{v}}\rho_{\mathrm{v}}VR.$$

We will apply the Laplace transform in the following form [3] to problem (1)-(6):

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